4/11/16 "To err is human; to forgive, divine." - Alexander Pope

HW: "Sequences" packet homework page #1, 2

AIM: What is a Sequence?

In Common Core Algebra I, you studied **sequences**, which are ordered lists of numbers. Sequences are extremely important in mathematics, both theoretical and applied. A **sequence** is formally defined as **a function that has as its domain the set the set of positive integers**, i.e. $\{1, 2, 3, \ldots, n\}$.

***Exercise* #1:** A sequence is defined by the equation $a(n) = 2n - 1$.

(a) Find the first three terms of this sequence, denoted by a_1, a_2, and a_3.

$a(1) = 2(1) - 1 = 1$

$a_1 = 1$

$a(2) = 2(2) - 1 = 3$

$a_2 = 3$

$a(3) = 2(3) - 1 = 5$

$a_3 = 5$

(b) Which term has a value of 53?

$a_n = 53$

what is the "n" value when $a(n) = 53$?

$a(n) = 2n - 1$

$53 = 2n - 1$

$+1 \qquad +1$

$54 = 2n$

$\frac{54}{2} = \frac{2n}{2}$

$n = 27$

(c) Explain why there will not be a term that has a value of 70.

$a_n = 70$

$a(n) = 2n - 1$

$70 = 2n - 1$

$+1 \qquad +1$

$71 = 2n$

$\frac{71}{2} = \frac{2n}{2}$

"n" has to be an integer. $35.5 = n$

Ex 1b)

$\underline{1}_{a_1}$ $\underline{3}_{a_2}$ $\underline{5}_{a_3}$ $\underline{7}_{a_4}$ $\underline{9}_{a_5}$ 11 13 . . - - - .

15 17 19 21 23 25 27

29 31 33 35 37 39 41

43 45 47 49 51 $\underline{53}_{a_{27}}$

Recall that sequences can also be described by using **recursive definitions**. When a sequence is defined recursively, terms are found by operations on previous terms.

start

***Exercise* #2:** A sequence is defined by the recursive formula: $f(n) = f(n-1)+5$ with $f(1) = -2$.

(a) Generate the first five terms of this sequence. Label each term with proper **function** notation.

$f(1) = -2$

$f(2) = f(2-1)+5$

$f(2) = f(1)+5$

$f(2) = -2+5$

$f(2) = 3$

$f(3) = f(2)+5$

$f(3) = 3+5$

$f(3) = 8$

$f(4) = f(3)+5$

$= 8+5$

$f(4) = 13$

$f(5) = f(4)+5$

$= 13+5$

$f(5) = 18$

$f(1) = -2$
$f(2) = 3$
$f(3) = 8$
$f(4) = 13$
$f(5) = 18$

(b) Determine the value of $f(20)$. Hint – think about how many times you have added 5 to -2.

$f(20)$ is the 20^{th} term.

Need to add 5 to -2 19 times.

$f(20) = -2+19(5)$

$= -2+95$

$= \boxed{93}$

***Exercise* #3:** Determine a recursive definition, in terms of $f(n)$, for the sequence shown below. Be sure to include a starting value.

5, 10, 20, 40, 80, 160, …

$$\boxed{f(n) = 2\cdot \underbrace{f(n-1)}_{\text{Previous Term}}}$$

Term Now

Starting Value:

$$\boxed{f(1)=5}$$

$$\underbrace{f(n)}_{\text{Term right now}} = \underbrace{f(n-1)}_{\text{Previous Term}} + 5$$

"To find any term, take the previous term and add 5".

n=1, 2, 3, 4

1. Given each of the following sequences defined by formulas, determine and label the first four terms. A variety of different notations is used below for practice purposes.

(a) $f(n)=7n+2$

n=1 $7(1)+2=\boxed{9}$

n=2 $7(2)+2=16$

n=3 $7(3)+2=23$

n=4 $7(4)+2=30$

(b) $a_n=n^2-5$

n=1 $1^2-5=-4$

n=2 $2^2-5=-1$

n=3 $3^2-5=4$

n=4 $4^2-5=11$

(c) $t(n)=\left(\frac{2}{3}\right)^n$

n=1 $\left(\frac{2}{3}\right)^1=\frac{2}{3}$

n=2 $\left(\frac{2}{3}\right)^2=\frac{4}{9}$

n=3 $\left(\frac{2}{3}\right)^3=\frac{8}{27}$

n=4 $\left(\frac{2}{3}\right)^4=\frac{16}{81}$

(d) $t_n=\frac{1}{n+1}$

n=1 $\frac{1}{1+1}=\frac{1}{2}$

n=2 $\frac{1}{2+1}=\frac{1}{3}$

n=3 $\frac{1}{3+1}=\frac{1}{4}$

n=4 $\frac{1}{4+1}=\frac{1}{5}$

2. Sequences below are defined recursively. Determine and label the **next** three terms of the sequence.

(a) $f(1)=4$ and $\boxed{f(n)=f(n-1)+8}$

add 8 to previous term

$f(1)=4$

$f(2)=4+8=12$

$f(3)=12+8=20$

$f(4)=20+8=28$

$\boxed{4, 12, 20, 28}$

(b) $a(n)=a(n-1)\cdot\frac{1}{2}$ and $a(1)=24$

multiply previous term by ½

$a(1)=24$

$a(2)=24\left(\frac{1}{2}\right)=12$

$a(3)=12\left(\frac{1}{2}\right)=6$

$a(4)=6\left(\frac{1}{2}\right)=3$

$\boxed{12, 6, 3}$

(c) $b_n=b_{n-1}+2n$ with $b_1=5$

previous term plus 2 times the term we are at.

$b_1=5$

$b_2=5+2(2)=9$

$b_3=9+2(3)=15$

$b_4=15+2(4)=23$

$\boxed{9, 15, 23}$

(d) $f(n)=2f(n-1)-n^2$ and $f(1)=4$

2 times previous term minus term now squared.

$f(1)=4$

$f(2)=2(4)-(2)^2=4$

$f(3)=2(4)-(3)^2=-1$

$f(4)=2(-1)-(4)^2=-18$

$\boxed{4, -1, -18}$

1 1 2 3 5 8 13 21 .....

if $f(n) = f(1) = 4$ Then $f(2-1) = f(1) = 4$

***Exercise* #4:** For the recursively defined sequence $t_n = (t_{n-1})^2 + 2$ and $t_1 = 2$, the value of t_4 is

(1) 18
(2) 38
(3) 456
(4) 1446

term now $= (\text{previous term})^2 + 2$

$t_1 = 2$

$t_2 = 2^2 + 2 = 6$

$t_3 = 6^2 + 2 = 38$

$t_4 = 38^2 + 2 = 1446$

***Exercise* #5:** One of the most well-known sequences is the Fibonacci, which is defined recursively using two previous terms. Its definition is given below.

$$f(n)=f(n-1)+f(n-2) \text{ and } f(1)=1 \text{ and } f(2)=1$$

Term Now

Previous Term

2 Terms ago

Generate values for $f(3)$, $f(4)$, $f(5)$, and $f(6)$ (in other words, then next four terms of this sequence).

$n=3$ $f(3)=f(2)+f(1)$

$f(3)=1+1$

$f(3)=2$

$f(4)=f(3)+f(2)$

$=2+1$

$f(4)=3$

$f(5)=f(4)+f(3)$

$=3+2$

$f(5)=5$

$f(1)=1$

$f(2)=1$

$f(3)=2$

$f(4)=3$

$f(5)=5$

$f(6)=8$

It is often possible to find algebraic formulas for simple sequence, and this skill should be practiced.

***Exercise* #6:** Find an algebraic formula $a(n)$, similar to that in *Exercise* #1, for each of the following sequences. Recall that the domain that you map from will be the set $\{1, 2, 3, \ldots, n\}$.

(a) 4, 5, 6, 7, ...

Pattern is adding 1

$a(n) = n+3$

$a(n) = 4 + 1(n-1)$

$= 4 + n - 1$

$a(n) = n+3$

(b) 2, 4, 8, 16, ...

Times 2

$\boxed{a(n) = 2^n}$

$a(n) = 2 \cdot 2^{n-1}$

(c) $5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \ldots$

$a(n) = \frac{5}{n}$

(d) $-1, 1, -1, 1, \ldots$

$a(n) = (-1)^n$

(e) 10, 15, 20, 25, ...

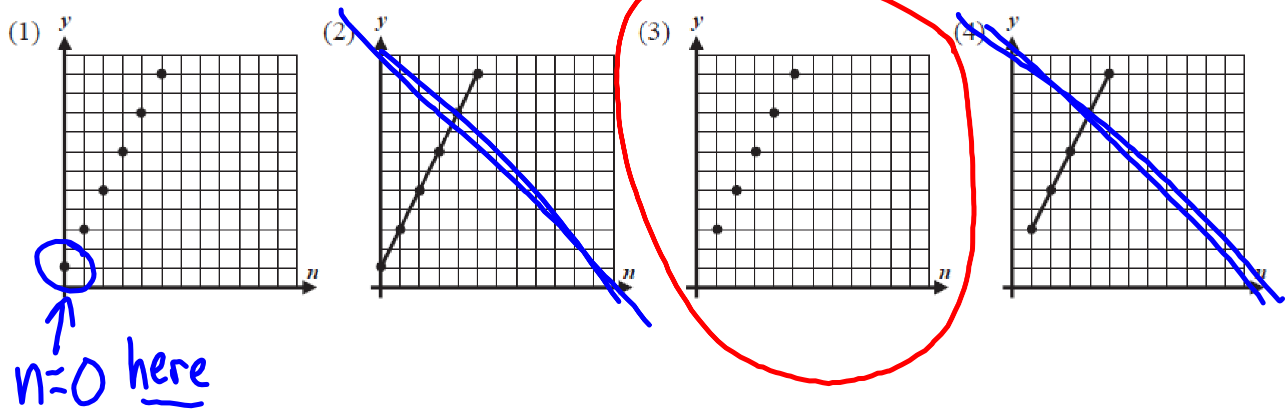
Pattern is add 5

$\boxed{a(n) = 5n+5}$

(f) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

$\boxed{a(n) = \frac{1}{n^2}}$

Exercise **#7:** Which of the following would represent the graph of the sequence $a_n = 2n+1$? Explain your choice.



Explanation: "n" has to be a positive integer